

CAHSEE contains 17 Measurement and Geometry items. As students relate their experiences from earlier classroom instruction in measurement and geometry to situations in their everyday lives, their knowledge and ability to apply this knowledge increase in depth and sophistication. To demonstrate understanding in this CAHSEE strand, students must be able to select and use appropriate units; estimate and calculate measurements for the length, area, and volume of geometric figures; understand scaling in scale drawings and how changes in linear dimension affect area and volume; and solve problems involving dimensional analysis and conversion from one unit to another.

To demonstrate achievement of the knowledge and skills in the measurement component of this strand, students should be able to use both metric and customary units of measurement for the following:

- determining the relationship between different units within the same system and converting from one unit to another within and between measurement systems
- using scale drawings and models to determine measurements of the original
- solving problems involving dimensional analysis for rates and other compound units
- relating the effect of changing the choice of a linear unit on the related square and cubic units for area and volume, respectively

The geometry component of this strand includes computing the perimeter, area, and volume of the most common 2- and 3-dimensional figures and using these common figures to estimate or compute the area of more complex objects.

To demonstrate acquisition of the knowledge and skills in the geometry component of the strand, students should be able to:

- describe, classify, and understand relationships between length, area, and volume among types of 2- and 3-dimensional objects.
- use coordinate geometry to represent and examine the properties of figures and their images under translation and reflection.
- understand and use the Pythagorean theorem.
- recognize and demonstrate understanding of congruence in terms of the sides and angles of 2-dimensional figures.

When CAHSEE items require students to use formulas, the formulas are provided within parentheses in the stem. All formulas the students may use, including the estimated value of  $\pi$ , will be provided *except* for the following:

- perimeter of a polygon (sum of the sides)
- circumference of a circle  $(C = 2\pi r, C = \pi d)$
- area of a triangle  $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares, base × height)
- volume of a rectangular prism (length × width × height)

The formula for finding the area of a nontraditional figure, such as a rhombus, will be provided.

In estimation problems, the approximate value of  $\pi$  that should be used to obtain the correct answer will be provided as part of the stem. Otherwise, the answer will include  $\pi$  (e.g.,  $12+3\pi$ ).

The ten California academic content standards covered by the CAHSEE Measurement and Geometry strand are discussed in the following pages.

Standard **7MG1.1** 

Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

## 2 test questions

## One millimeter is—

- A  $\frac{1}{1000}$  of a meter.
- $\mathbf{B} = \frac{1}{100}$  of a meter.
- C 100 meters.
- **D** 1000 meters.

M00276

Comparing units and computing the effect of changing units are essential skills for acquiring knowledge in mathematics and science disciplines. CAHSEE questions for this standard require students to convert between two units of measurement within the same system or between two different systems of measurement. Conversion formulas are provided for test questions that require the student to convert between less commonly used units (such as pints to gallons), systems of measurement, square units, and cubic units.

# Sample Test Question

The correct answer is choice A. Students must know that to make this conversion, 1 millimeter

is equal to  $\frac{1}{1000}$  of a meter.

Analysis of Distractors

A: correct answer

Distractor B: used  $\frac{1}{100}$  as the conversion factor

Distractor C: used 100 as the conversion factor

Distractor D: used 1000 as the conversion factor

Standard 7MG1.2

Construct and read drawings and models made to scale.

1 test question

The actual width (w) of a rectangle is 18 centimeters (cm). Use the scale drawing of the rectangle to find the actual length (l).

**A** 6 cm

**B** 24 cm

**C** 36 cm

**D** 54 cm

M02087

The ability to move back and forth between a scale drawing or model and a real object is essential for understanding representation as well as proportional reasoning. Questions on the CAHSEE focus on the second component of this standard, reading scale drawings and models. Students will be asked to read and interpret drawings and scale models. Students may also be asked to apply given measurements to determine the scale of a figure.

#### Sample Test Question

The correct answer is choice D. Students must recognize that the ratio between the width of the scale drawing and the actual width of the rectangle is the same as that between the length of the scale drawing and the actual length. Students may use a variety of approaches to solve the

problem, including setting up a proportion similar to  $\frac{1.2}{18} = \frac{3.6}{l}$  and solving for l.

Analysis of Distractors

Distractor A: used an incorrect relationship between the parts of the proportion,  $(18 \div 3.6) \cdot 1.2 = 6$ 

Distractor B: subtracted the width from the length and multiplied by 10

Distractor C: used 3.6 • 10 rather than 3.6 • 15

D: correct answer

Standard **7MG1.3** 

Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

2 test questions

# Sixty miles per hour is the same rate as which of the following?

- **A** 1 mile per minute
- **B** 1 mile per second
- **C** 6 miles per minute
- **D** 360 miles per second

M02473

Dimensional analysis is a critical skill in physical science, engineering, and the social sciences. Students should be familiar with the rates named in the standard as well as other commonly used measures (e.g., kilowatt-hours, foot-pounds, acre-feet, person-hours). Questions on the CAHSEE for this standard focus on all three of its components: measures as rates, measures as products, and reasonableness of results.

#### Sample Test Question

The correct answer is choice A. Students may reason that another way to express the given rate is 60 miles per 60 minutes, since 1 hour is equivalent to 60 minutes. The expression is 60 miles/hour is equivalent to 60 miles/60 minutes and that since  $60 \div 60 = 1$ , the rate is equivalent to 1 mile/minute. Students should also be encouraged to evaluate the answer choices in the question for their reasonableness as a restatement of this real-world rate.

Analysis of Distractors

A: correct answer

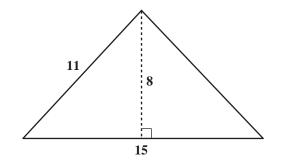
Distractor B: equated 1 hour with 60 seconds

Distractor C: divided 60 miles per hour by 10, rather than by 60

Distractor D: multiplied 60 by 6

Standard 7MG2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

## 3 test questions



## What is the area of the triangle shown above?

- **A** 44 square units
- **B** 60 square units
- C 88 square units
- **D** 120 square units

M00101

All students should know how to compute the area and volume of basic figures and also how to apply basic formulas in many areas to solve problems. This standard requires students to find the perimeter and area of 2-dimensional figures and the surface area and volume of 3-dimensional figures. The figures tested on the CAHSEE include parallelograms, trapezoids, triangles, circles, prisms, and cylinders. Additional items for this standard may ask students to find the length of a side of a figure when given the length of a known side, perimeter, area, or a combination of these values. Students are required to know the following formulas:

- perimeter of a polygon (sum of the sides)
- circumference of a circle  $(C = 2\pi r, C = \pi d)$
- area of a triangle  $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares, base  $\times$  height)
- volume of a rectangular prism (length × width × height)

These formulas will not be given with the test questions. The estimated value of  $\pi$  will be given unless  $\pi$  is included in the answer choices.

## Sample Test Question

The correct answer is choice B. Students should know the area formula  $A = \frac{1}{2}bh$  and use it to determine that the height is 8 units and the base is 15 units. The area is 60 square units.

Distractor A: used 11 as the base, rather than 15

B: correct answer

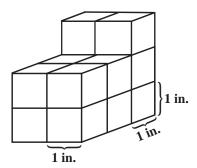
Distractor C: used 11 as the base and did not take  $\frac{1}{2}$  of the product of the base and the height

Distractor D: did not take  $\frac{1}{2}$  of the product of the base and the height

Standard 7MG2.2
Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

#### 2 test questions

One-inch cubes are stacked as shown in the drawing below.



#### What is the total surface area?

**A** 19 in.<sup>2</sup>

**B** 29 in.<sup>2</sup>

C 32 in.<sup>2</sup>

**D**  $38 \text{ in.}^2$ 

M02812

This standard emphasizes the development of problem-solving skills with visual tools. CAHSEE questions for this standard focus on both of its components. The first component requires students to identify the measurable and/or computable parts of a shape or structure. This identification task is largely dependent on the ability to visualize the familiar geometric structures that make up a more complex figure. Where estimation is required, students may be given dimensions of known shapes, scale and proportion, or grids in the visual prompt. The second component requires students to estimate and/or compute the area of the subdivided portions of a figure. Frequently, the dimensions of the component parts are not given directly by labeled measurements but must be determined by such means as adding or subtracting lengths or extending lines.

#### Sample Test Question

The correct answer is choice D. The foreground portion of the object contains a 2 by 2 by 2-cube grouping, and the background contains a 1 by 2 by 3-cube grouping. To find the surface area, students must visually assemble the surface of the individual cubes that make up the entire object and recognize that the object has 8 planar faces: right, rear, left, front, bottom, front top, rear face, and rear top. The surface areas of each corresponding face are 7, 6, 7, 4, 6, 4, 2, and 2 square inches, and the sum of the areas of these faces is 38 square inches.

Distractor A: included only half of the exterior faces

Distractor B: did not include the left or right face and either the rear face or rear top

Distractor C: did not include the bottom or rear of the figure

D: correct answer

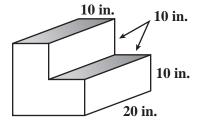
Standard 7MG2.3

Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor.

1 test question

The short stairway shown below is made of solid concrete. The height and width of each step is 10 inches (in.). The length is 20 inches.



What is the volume, in cubic inches, of the concrete used to create this stairway?

**A** 3000

**B** 4000

**C** 6000

**D** 8000

M02990

This standard contains two components. The first component focuses on the students' ability to compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

The second component focuses on the students' understanding of the results of multiplying the lengths of all dimensions by a scale factor, i.e., the surface area is multiplied by the square of the scale factor or the volume is multiplied by the cube of the scale factor. Items addressing the scale factor component of this standard may include two-dimensional objects.

#### Sample Test Question

The correct answer is choice C. Students should find the volume of the larger rectangular prism by multiplying 10 times 20 times 20. They should then find the volume of the smaller rectangular prism by multiplying 10 times 10 times 20. The sum of these 2 volumes gives the correct answer.

Distractor A: found the volume of the larger prism by multiplying 10 times 10 times 20 and the volume of the smaller prism by multiplying 10 times 10

Distractor B: found the volume of only the larger prism

C: correct answer

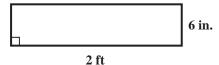
Distractor D: doubled the volume of the larger prism

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Standard 7MG2.4
Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversion between units (1 square foot = 144 square inches or  $\begin{bmatrix} 1 & \text{ft}^2 \end{bmatrix} = \begin{bmatrix} 144 & \text{in}^2 \end{bmatrix}$ , 1 cubic inch is approximately 16.38 cubic centimeters or  $\begin{bmatrix} 1 & \text{in}^3 \end{bmatrix} = \begin{bmatrix} 16.38 & \text{cm}^3 \end{bmatrix}$ ).

The width of the rectangle shown below is 6 inches (in.). The length is 2 feet (ft).



What is the area of the rectangle in square inches?

- **A** 12
- **B** 16
- **C** 60
- **D** 144

M03243

Unit conversions for area and volume have practical applications in students' lives. CAHSEE questions for this standard require students to relate the changes in measurement with a change of scale to the units used and to convert between units. The emphasis is on the relationship between linear units for distance, square units for area, and cubic units for volume. Conversion formulas will be provided for linear measures except for inches to feet to yards or conversions within the metric system.

## Sample Test Question

1 test question

The correct answer choice is D. Students should convert 2 feet to 24 inches and multiply 6 and 24 to get the area of the rectangle.

Analysis of Distractors

Distractor A: multiplied 6 and 2

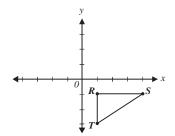
Distractor B: found the perimeter of the rectangle using the given values without making the conversion of feet to inches

Distractor C: made the conversion of feet to inches, but then found the perimeter of the rectangle

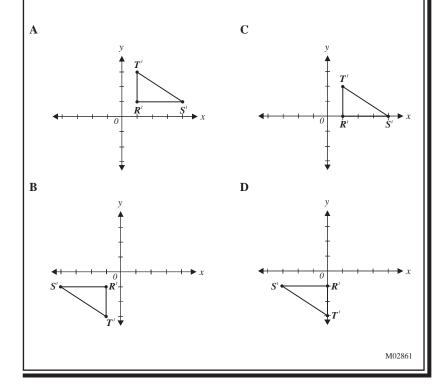
D: correct answer

Standard 7MG3.2
Understand and use
coordinate graphs to
plot simple figures,
determine lengths and
areas related to them,
and determine their
image under translations
and reflections.

## 2 test questions



Which of the following triangles R'S'T' is the image of triangle RST that results from reflecting triangle RST across the y-axis?



Understanding the *xy* coordinate system is an essential step in developing important thinking and problem-solving skills. Visualizing and manipulating objects are useful tools, not only in geometry but also in other disciplines.

CAHSEE questions for this standard assess students' understanding of all components of the standard: plotting simple figures with ordered pairs, determining lengths and areas from plotted figures, and finding images following transformations by translations and reflections.

#### Sample Test Question

The correct answer is choice B. Students must recognize the reflective correspondence between points R and R', S and S', and T and that triangle R'S'T' is the reflective image, across the y-axis, of triangle RST.

Distractor A: reflection of triangle *RST* across the *x*-axis

B: correct answer

Distractor C: reflection of triangle *RST* across the *x*-axis and its translation by 1 unit down or its reflection across  $y = -\frac{1}{2}$ 

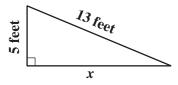
Distractor D: reflection of triangle RST across the y-axis and its translation by 1 unit to the right

Standard 7MG3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

# 2 test questions

What is the value of *x* in the right triangle shown below?



- A 8 feet
- **B** 12 feet
- C 18 feet
- **D** 23 feet

M03181

The Pythagorean theorem is important for its problem-solving function as well as its role as a bridge between geometry and algebra. CAHSEE questions assessing this standard address the following components of the standard: using the Pythagorean theorem to find the length of the missing base, altitude, or hypotenuse of a right triangle and using the Pythagorean theorem to find lengths of line segments in figures other than triangles.

#### Sample Test Question

The correct answer is choice B. Students should use the Pythagorean theorem  $5^2 + x^2 = 13^2$  to obtain the correct answer of 12.

#### Analysis of Distractors

Distractor A: the difference of the hypotenuse and altitude

B: correct answer

Distractor C: the sum of the hypotenuse and altitude

Distractor D: the sum of the hypotenuse and twice the altitude

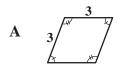
Standard 7MG3.4

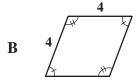
Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

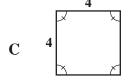
## 1 test question

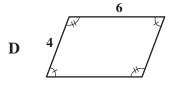


Which figure is congruent to the figure shown above?









M00020

CAHSEE questions for this standard require students to discriminate between figures that are or are not congruent to a given figure. Students may also be asked to identify specific reasons to support the assertion that two figures are congruent. Relationships between sides and angles of figures are significant because they may indicate the presence or absence of congruence by axiomatic reasoning, such as side-angle-side congruence. Items in this standard will not involve the use of acronyms for side and angle relationships of congruent figures (e.g., SAS, SSS).

### Sample Test Question

The correct answer is choice B. Students should understand that two polygons are congruent if all corresponding sides and angles are congruent. Reasoning from this basis, students should recognize that the figure in choice B is congruent to the given figure because of the correspondence with sides of length 4 and angles with two marks. Students should also understand that because both figures are rhombi with non-congruent adjacent angles, all four sides are congruent and the opposite angles are congruent.

Analysis of Distractors

Distractor A: a similar, but not congruent figure

B: correct answer

Distractor C: a square

Distractor D: a parallelogram with non-congruent adjacent sides